MATH 317 — Midterm 1 — 50 minutes

03 October 2025

- The test consists of 8 pages and 5 questions worth a total of 50 marks.
- This is a closed-book examination. **None of the following are allowed**: documents, formula sheets, electronic devices of any kind (including calculators, cell phones, etc.)
- No work on this page will be marked.
- Fill in the information below before turning to the questions.

Student number								
Section								
Name	. . . . .							
Signature								

NOTICE: Look at both sides of every sheet.
Some questions start on the back.

Turn off and put away all cell phones, pagers, alarms, etc., before the exam begins. Any such device that disrupts the exam will be confiscated.

Rules Governing Formal Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.

2. Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.

3. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

4. Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.

5. Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:

(a) speaking or communicating with other candidates, unless otherwise authorized;

(b) purposely exposing written papers to the view of other candidates or imaging devices;

(c) purposely viewing the written papers of other candidates;

(d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,

(e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s) (electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).

6. Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.

7. Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Additional instructions

- **For every question, write your answers in *the space provided for that question.***
- **Use both sides of every page.**
- **If you need more work space, request an official Extra Sheet from an invigilator. On any Extra Sheet, write your identification on every side you want marked. (You may use different sides for different questions.)**
- **Work on this page will not be marked.**

Selected Formulas

Vector identities:

$$\mathbf{u}\times(\mathbf{v}\times\mathbf{w}) = (\mathbf{w}\bullet\mathbf{u})\mathbf{v} - (\mathbf{v}\bullet\mathbf{u})\mathbf{w},$$
$$\mathbf{u}\bullet(\mathbf{v}\times\mathbf{w}) = (\mathbf{u}\times\mathbf{v})\bullet\mathbf{w}.$$

Frenet-Serret: For $\mathbf{r}=\mathbf{r}(t)$, $\dfrac{d\widehat{\mathbf{T}}}{dt} = v\kappa\widehat{\mathbf{N}}$, $\dfrac{d\widehat{\mathbf{N}}}{dt} = v\tau\widehat{\mathbf{B}} - v\kappa\widehat{\mathbf{T}}$, $\dfrac{d\widehat{\mathbf{B}}}{dt} = -v\tau\widehat{\mathbf{N}}$.

Classical Newtonian Gravitation: $\mathbf{F} = -\dfrac{GMm}{r^2}\left(\dfrac{\mathbf{r}}{r}\right)$; $G = 6.67\times10^{-11}\,\dfrac{\mathrm{Nm}^2}{\mathrm{kg}^2}$.

Polar Coordinates: At $x = r\cos\theta$, $y = r\sin\theta$ (with $z=0$ for 3D), local unit vectors are $\widehat{r} = \cos\theta\mathbf{i} + \sin\theta\mathbf{j}$ and $\widehat{\theta} = -\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$. (In 3D, we have $\widehat{r}\times\widehat{\theta} = \widehat{z} = \mathbf{k}$.)

In a time-varying situation, $\dfrac{d\widehat{r}}{dt} = \dot\theta\widehat{\theta}$, $\dfrac{d\widehat{\theta}}{dt} = -\dot\theta\widehat{r}$.

Thus $\mathbf{r} = r\widehat{r}$ leads to $\mathbf{v} = \dot r\widehat{r} + r\dot\theta\widehat{\theta}$, $\mathbf{a} = \left(\ddot r - r\dot\theta^2\right)\widehat{r} + \left(r\ddot\theta + 2\dot r\dot\theta\right)\widehat{\theta}$.

Kepler I: $r = \dfrac{\ell}{1+\varepsilon\cos\theta}$; $\ell = \dfrac{h^2}{GM}$. ("Orbits are ellipses, ...")

Kepler II: $A(t_1) - A(t_0) = \frac{1}{2}h^2(t_1 - t_0)$; $h = r^2\dot\theta$ is constant. ("Equal areas in equal times.")

Kepler III: $T^2 = \dfrac{4\pi^2}{GM}a^3$. ("Period squared is proportional to major axis cubed.")